THE detailed explanation concerning the operation of UNIT "A" is continued in this month's article, with further practical examples.

We resume by considering the use of the operational amplifier as an integrator.

An operational amplifier will be handling time as well as voltage when acting as an integrator, so some means must be found of inserting intervals of time onto the computer. One method is to employ external oscillators to provide known functions of time in terms of frequency. An input to an integrator might consist of a steady d.c. voltage which is switched on for a time t (step function or square wave), or alternatively, a sinusoidal voltage of frequency f and period $1/f$.

If a graph is drawn of the resulting integrator output function, and this is the form that answers to problems involving change or motion will usually take, the X axis of the graph will be calibrated in intervals of time, with voltage on the Y axis. It follows that an oscilloscope, which also uses time on the X axis and voltage on the Y axis, can provide a convenient form of output display, especially when an integrator is operating at high speed.

The operational amplifier is converted to an integrator when a capacitor C_f is inserted, in place of a resistor, in the feedback path; see Fig. 5.1. When an input voltage $-E_{in}$ is applied to the integrator by means of a simple switch S for a time t, the output E_o will take the form of an increasing ramp voltage proportional to t with slope

$$-E_{in}\ \frac{1}{R_{in}C_f}$$

Note that the operational amplifier will continue to invert an input voltage even when used as an integrator.

THE INTEGRATOR IN EQUATION SOLVING

The electronic analogue computer does provide a powerful technique for obtaining rapid solutions to problems involving calculus, which cannot be equalled either by numerical methods or by a digital computer.

If differentiation and integration are regarded as straightforward mathematical operations, it will be found that the terms of, say, a second order differential equation can be manipulated on the computer in much the same way as the terms of a "steady state" algebraic equation.

For example, when an equation term y is differentiated against time its derivative dy/dt is obtained, and a second differentiation yields the second derivative d^2y/dt^2. The reverse process is where integration of the second derivative d^2y/dt^2 produces the first derivative dy/dt, and another integration gives y as the result.

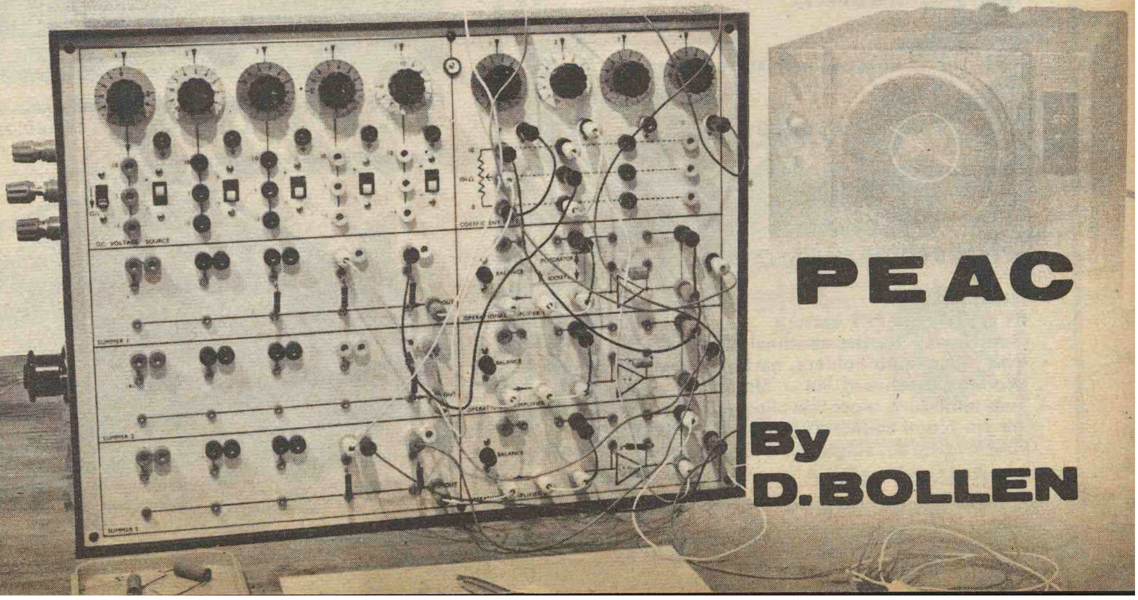
Fig. 5.2 shows how a simple integrator can handle equation terms. Combined operations are made possible by cascading integrators, while using coefficient potentiometers and computing component ratios for summation, multiplication, and division (Fig. 4.1).

The process of differentiation, although feasible if care is taken, is generally avoided on analogue computers because it gives rise to unstable operational amplifier configurations, but this imposes only a slight limitation since integration can be employed—in the majority of cases—in place of differentiation.

INTEGRATOR ACCURACY

The transfer accuracy of an operational amplifier, when it is used as an integrator, will be theoretically limited by its finite value of open-loop gain. However,

ANALOGUE COMPUTER



PE AC

By D. BOLLEN

TABLE 5.1

C_f	R_{in}	t
$1\mu F$	$100k\Omega$ $10k\Omega$	2·8sec 800ms
$0{\cdot}1\mu F$	$100k\Omega$ $10k\Omega$	280ms 80ms
$0{\cdot}01\mu F$	$100k\Omega$	28ms

Maximum value of t for an error of 1%

the situation is much more complicated than with, for example, a summing amplifier (Fig. 3.8) since the amplifier error can no longer be defined in terms of the simple relationship between closed-loop and open-loop gains.

As a guiding principle, integrating amplifiers may have very large values of closed-loop gain provided that the time t of an input function remains small. Closed-loop integrator gains of 1,000 or more are not uncommon in transistor computers, since low voltages and low impedances discourage the use of computing resistors of more than 100 kilohm, and capacitors of more than $1\mu F$ are too bulky. Table 5.1 is calculated for UNIT "A" amplifiers, and sets out the maximum allowable interval t for selected values of C_f and R_{in}, where the amplifier transfer error must not exceed one per cent.

Errors due to unwanted drift voltages also become significant when t is long and C_f is small. The greatest care must be exercised when zero-setting integrators to eliminate offset voltages, for good accuracy at long time intervals. Also, the computer should not be subjected to fluctuations of ambient temperature when computations cover several hours of integrator use.

COMPUTING CAPACITORS

The computing capacitors used for PEAC will normally lie within the range $0{\cdot}01$–$1\mu F$, and the three values most commonly employed are $0{\cdot}01\mu F$, $0{\cdot}1\mu F$, and $1\mu F$. Polystyrene is the preferred capacitor dielectric, for high insulation resistance, but polyester makes an acceptable second best. Mica, paper, and ceramic capacitors should be avoided.

Small value polystyrene capacitors of ± 1 per cent and ± 2 per cent tolerance are easily obtained, but $0{\cdot}1\mu F$ and $1\mu F$ precision components are rare and expensive. To get around this difficulty, the bridge circuit of Fig. 5.3 was devised to allow computing capacitors to be made up from specially selected low cost ± 20 per cent capacitors.

The circuit of Fig. 5.3 can be constructed in breadboard form on Veroboard or s.r.b.p., with miniature sockets to take C_x and R1. If an audio signal generator is not available to supply the bridge with about 10V r.m.s. at 1kHz, a signal could be obtained from a transistor multivibrator powered by the 25V computer power supply. Headphones serve to detect the null point when the bridge is in balance, and should have an impedance of about 2 kilohms.

The method of making up a computing capacitor of, say, $1\mu F$ is as follows. A capacitor panel of plain or perforated s.r.b.p. is fitted with small turret tags as in Fig. 5.4. A ± 20 per cent capacitor of about $0{\cdot}68\mu F$ is wired into position on the capacitor panel before it is plugged into the bridge C_x sockets, and a 1 kilohm

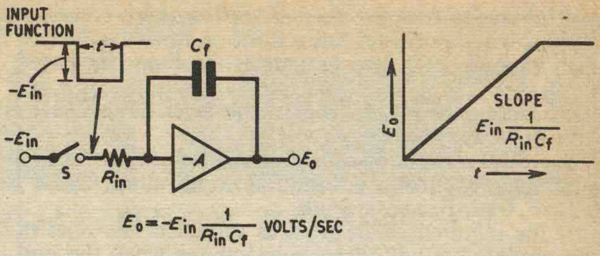


Fig. 5.1. The operational amplifier as an integrator

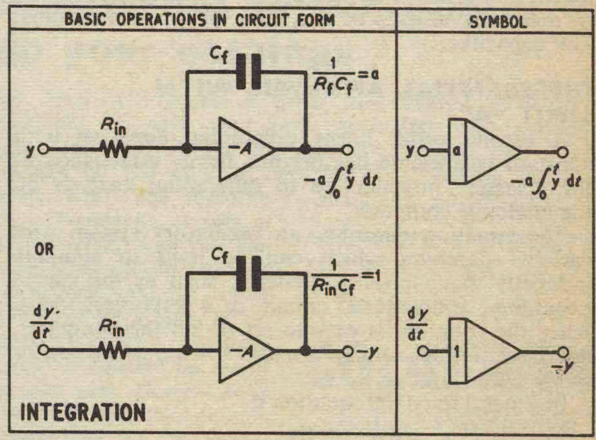


Fig. 5.2. The handling of equation terms by a simple integrator

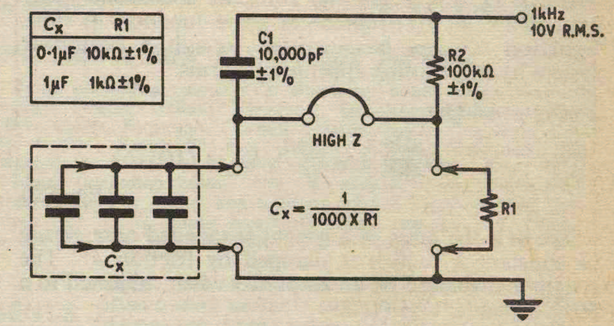


Fig. 5.3. Bridge circuit used for making up computing capacitors

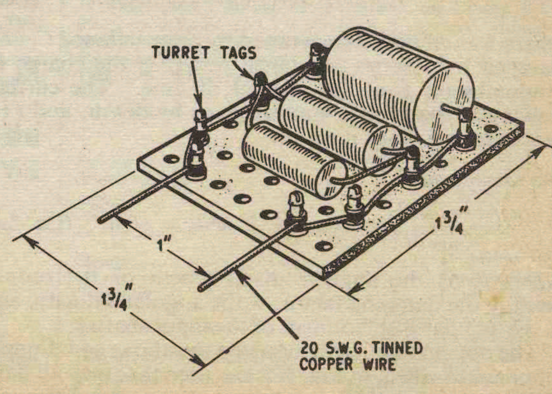


Fig. 5.4. Computing capacitor plug-in panel

resistor is inserted for R1. Assorted polystyrene or good quality polyester capacitors of lower value are then temporarily connected across the capacitor panel to increase C_x by small increments, while listening on the headphones for a drop in the level of the 1kHz tone as C_x approaches 1μF.

A typical computing capacitor might finally consist of a parallel combination of the following values, 0·68μF, 0·22μF, 0·02μF, and 0·005μF.

If the required value of C_x is exceeded, the note in the headphones will increase in volume when the null point is passed. Allow capacitors to cool off after soldering, and before making a measurement, as heat can cause a temporary or permanent change in capacitance. With the Fig. 5.3 bridge circuit it is possible to detect increments of less than 0·01μF in a nominal 1μF capacitor.

DIFFERENTIAL ANALYSIS WITH UNIT "A"

A second order linear differential equation with constant coefficients has become firmly established as the "classic" introduction to differential analysis on the analogue computer.

The equation describes an oscillatory system with variable damping which can be used to simulate indirectly many physical systems, such as the spring pendulum, a tuned *LC* circuit, or a servomechanism. Also, the equation is easy to set up on the computer, and does not necessarily demand the use of integrator mode switching.

In general form the equation is,

$$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = f(t) \qquad \text{(Eq. 5.1)}$$

where *a*, *b*, and *c* are the constant coefficients, *y* is unknown, and $f(t)$ represents some function of time. Equation 5.1 can be rewritten to suit a particular system by substituting appropriate terms.

Spring pendulum

$$m\frac{d^2y}{dt^2} + \mu\frac{dy}{dt} + ky = f(t) \qquad \text{(Eq. 5.2)}$$

where *m* is the mass of a weight suspended on a spring of constant *k*, which is damped by friction *μ*. The weight is displaced by an amount *y* when subjected to a force dependent on $f(t)$.

Tuned LC circuit

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = f(t) \qquad \text{(Eq. 5.3)}$$

where *L* is an inductance tuned by a capacitance *C*, and damped by a series resistance *R*. *Q* is the charge in coulombs on *C* at any instant of time. The current flowing in the tuned circuit is given by dQ/dt, and $f(t)$ represents an input function.

Servomechanism

$$\frac{d^2\theta_o}{dt^2} + 2\zeta\omega\frac{d\theta_o}{dt} + \omega^2\theta_o = \omega^2\theta_i \qquad \text{(Eq. 5.4)}$$

where θ_o is the angular displacement of the output shaft, ζ the damping factor, ω the angular velocity, and θ_i the angular displacement of the input shaft.

The obvious similarity between the above equations is emphasised when, in Fig. 5.5, it is seen that they all have virtually the same problem layout on the computer. Furthermore, as the computer will allow operation at almost any fraction or multiple of real time, a spring pendulum and a tuned *LC* circuit can be simulated simultaneously, and interesting electro-mechanical parallels can be seen to exist between the properties of inductance and mass, resistance and friction, and capacitance and elasticity.

The only real difference between the analogous behaviour of a weight on a spring, a servo shaft, and a tuned *LC* circuit is that the *LC* combination will normally resonate at a much higher frequency.

PROBLEM EXAMPLE 3. TUNED CIRCUIT ANALYSIS

UNIT "A" will simulate any series tuned circuit by solving Equation 5.2, and will give answers in the form of a.c. meter readings or oscillograms. Tuned circuits resonating in the MHz region are catered for by slowing down the problem to some convenient decadal fraction of real time, so that a simulated circuit on the computer which is, for example, resonating at 300Hz, will serve as a model for a real circuit resonating at 30MHz, with suitable rescaling of *L*, *C*, and *t*.

To initially determine the relative values of *L*, *C*, *R*, voltage *V*, and current *I*, without too much paperwork, it is helpful to start with a representative tuned circuit which allows computer operation in real time, at frequencies convenient for display by an a.c. voltmeter or an oscilloscope. 50Hz is a good frequency to employ as a datum because it can be readily obtained from the mains supply, and rounded values of $L = 1$H and $C = 10\mu$F will also offer resonance at 50Hz.

Taking the circuit of Fig. 5.6a as a starting point, from the knowledge that a series tuned circuit will exhibit an impedance equal to *R* at resonance, the r.m.s. current flow at 50Hz will be E_i/R, or 20mA when $E_i = 2$V r.m.s. and $R = 100$ ohms.

It is necessary to rearrange the basic equation, Equation 5.2, for the computer by dividing through by *L*, and solving for the second derivative.

$$\frac{d^2Q}{dt^2} = -\frac{R}{L}\frac{dQ}{dt} - \frac{1}{LC} + \frac{f(t)}{L} \qquad \text{(Eq. 5.5)}$$

Substituting known values from Fig. 5.6a,

$$\frac{d^2Q}{dt^2} = \frac{100R}{1\text{H}}\frac{dQ}{dt} - \frac{1}{1\text{H} \times 10^{-5}C}Q + \frac{f(t)}{1\text{H}} \qquad \text{(Eq. 5.6)}$$

$f(t)$ in the present case represents a sine wave input of 2V r.m.s. In other circumstances the input function could be a square wave of amplitude E_{in} and period $2t$.

Equation 5.6 is solved on the computer by successive integration. Looking at the symbolised diagram of Fig. 5.6b, it can be seen that there are two closed-loops, one linking the output of OA1 via CP1 to OA1/Input 1, and the other passing through OA1, OA2, and OA3, via CP2, and thence back to OA1/Input 3. The coefficient of CP1 will be multiplied by the gain factor associated with OA1/Input 1. CP2 coefficient is multiplied by the product of gains OA1/Input 3, OA2, and OA3, i.e. 1,000 × 100 × 1 = 100,000.

d^2Q/dt^2, obtained from the sum of the voltages present at the inputs of OA1, is initially assumed to be present. After one integration OA1 provides an output dQ/dt, and from this all the terms on the right hand side of Equation 5.6 are assembled. So, dQ/dt is multiplied by $R/L = 100$, using CP1 set for a coefficient of 0·1, and is taken back to OA1/Input 1 where it is then added to $f(t)/L = 2$V r.m.s.

Moving in the other direction on the symbolised diagram of Fig. 5.6b, dQ/dt is integrated by OA2 to obtain $+Q$. Inverting amplifier OA3 changes the sign of Q before passing it on for multiplication by $1/LC = 100{,}000$ (CP2 coefficient of 1). $-(1/LC)Q$ is then added, at OA1/Input 3, to

$$-\frac{R}{L}\frac{dQ}{dt}+\frac{f(t)}{L}$$

and the sum of all OA1 input voltages yields the required d^2Q/dt^2. Because there are two closed-loops in the computer set-up the equation will be self-enforcing.

Routine. Switch on UNIT "A" power supply and allow a warm-up time of at least 15 minutes. Ensure that the three operational amplifiers are disconnected from their summer networks, and have no feedback components. Apply 10V d.c. voltmeter leads to OA1/SK13 and an earth socket, and zero-set OA1 for an output voltage of less than ± 1V from the back of the UNIT "A" box, by means of VR1 (Fig. 3.7). Repeat for OA2 and OA3.

Set up the problem according to the patching circuit of Fig. 5.6b, but omit the feedback capacitors and the patching link between OA3/SK13 and CP2/SK1. Set CP1 dial to approximately "1". Connect the voltmeter to miniature socket OA1/SK6 (Fig. 2.9) and zero-set OA1 again, but this time using the front panel control VR15.

Next, zero-set OA2 using VR16, and OA3 using VR17. Insert 0·1μF computing capacitors into OA1/SK11 and SK12, and OA2/SK11 and SK12, and make good the link between OA3 output and CP2. Set CP2 for a dial reading of "10". Apply the voltmeter to OA2/SK7 and zero-set the complete assembly of amplifiers by adjustment of VR15(OA1) only.

The problem layout will now be ready for dynamic checks and should not need to be re-zeroed for several hours if UNIT "A" is being operated in stable ambient temperature conditions.

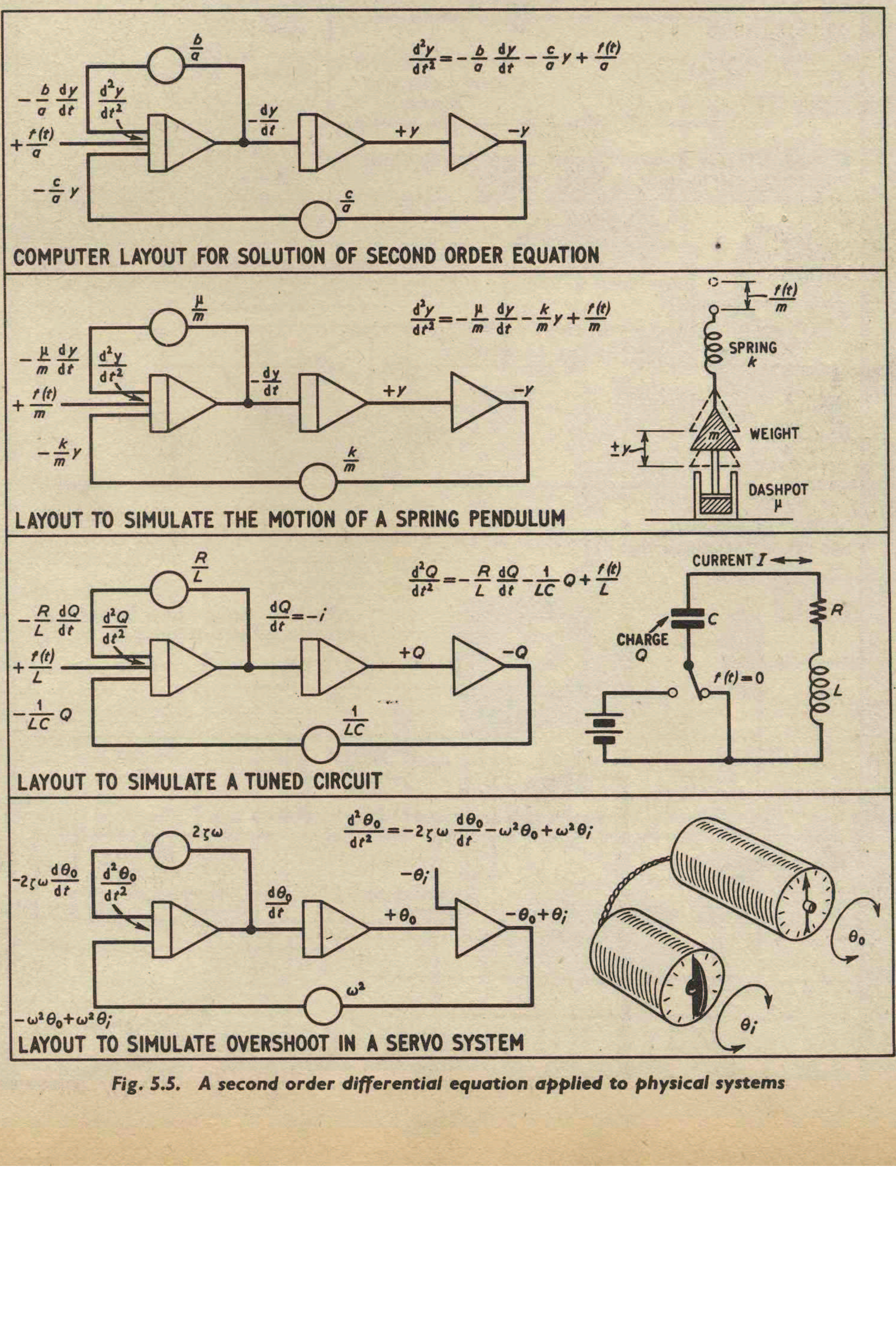


Fig. 5.5. A second order differential equation applied to physical systems

TABLE 5.2

SHOWING HOW COMPUTER OPERATING FREQUENCIES ARE RELATED TO CP2 SETTING AND AMPLIFIER CLOSED-LOOP GAINS

Resonant Frequency f	Typical Values L	C	CP2 Coefficient	$\frac{1}{LC}$	Amplifier Gains OA1 Input 3	OA2	OA3
0·05Hz to	1,000H	10,000μF	0·1	0·1	10	10	0·1
0·5Hz	100H	1,000μF	1·0	10	10	10	0·1
5Hz to	10H	100μF	0·01	10^3	1,000	100	1·0
50Hz	1H	10μF	1·0	10^5	1,000	100	1·0
500Hz	100mH	1μF	1·0	10^7	1,000	1,000	10
1kHz	100mH	0·2μF	1·0	5×10^7	1,000	1,000	50

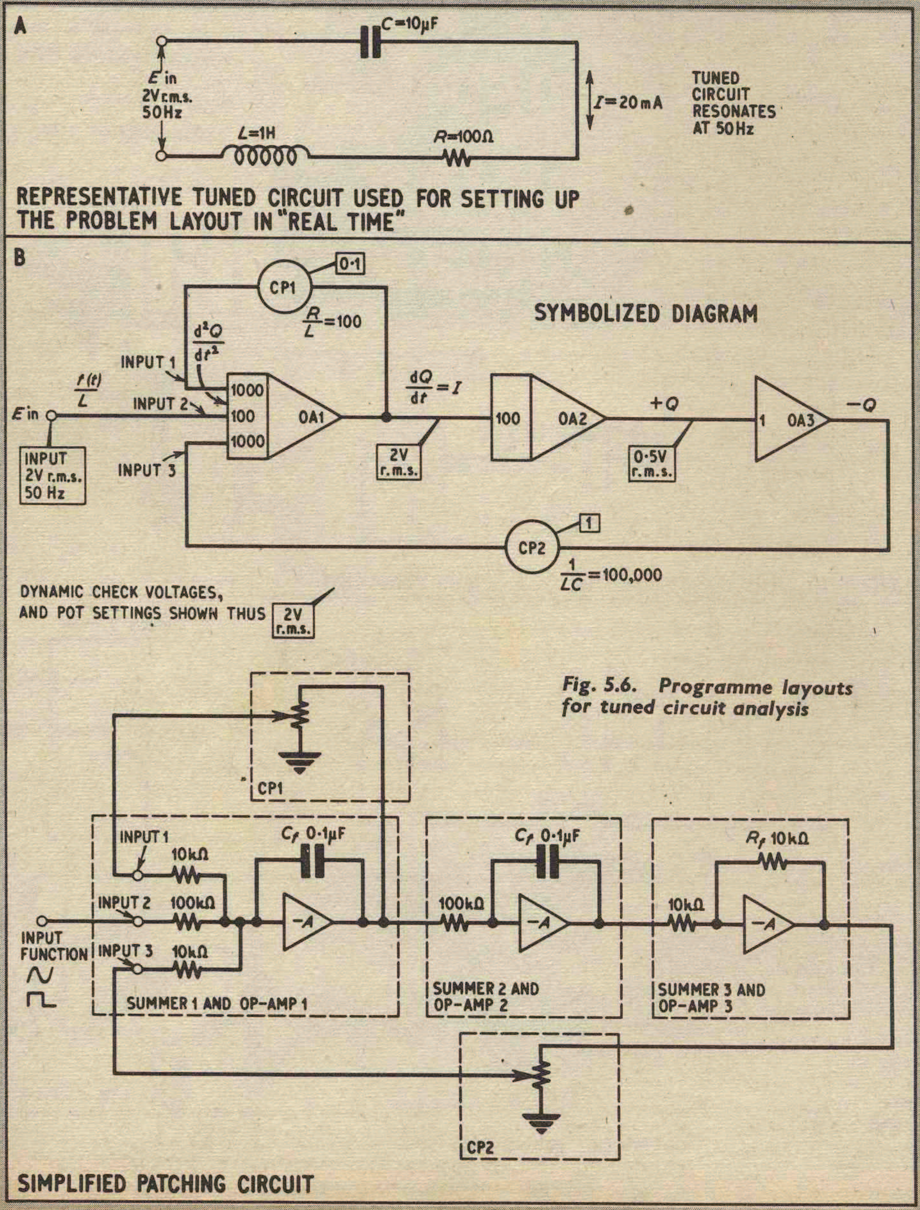


Fig. 5.6. Programme layouts for tuned circuit analysis

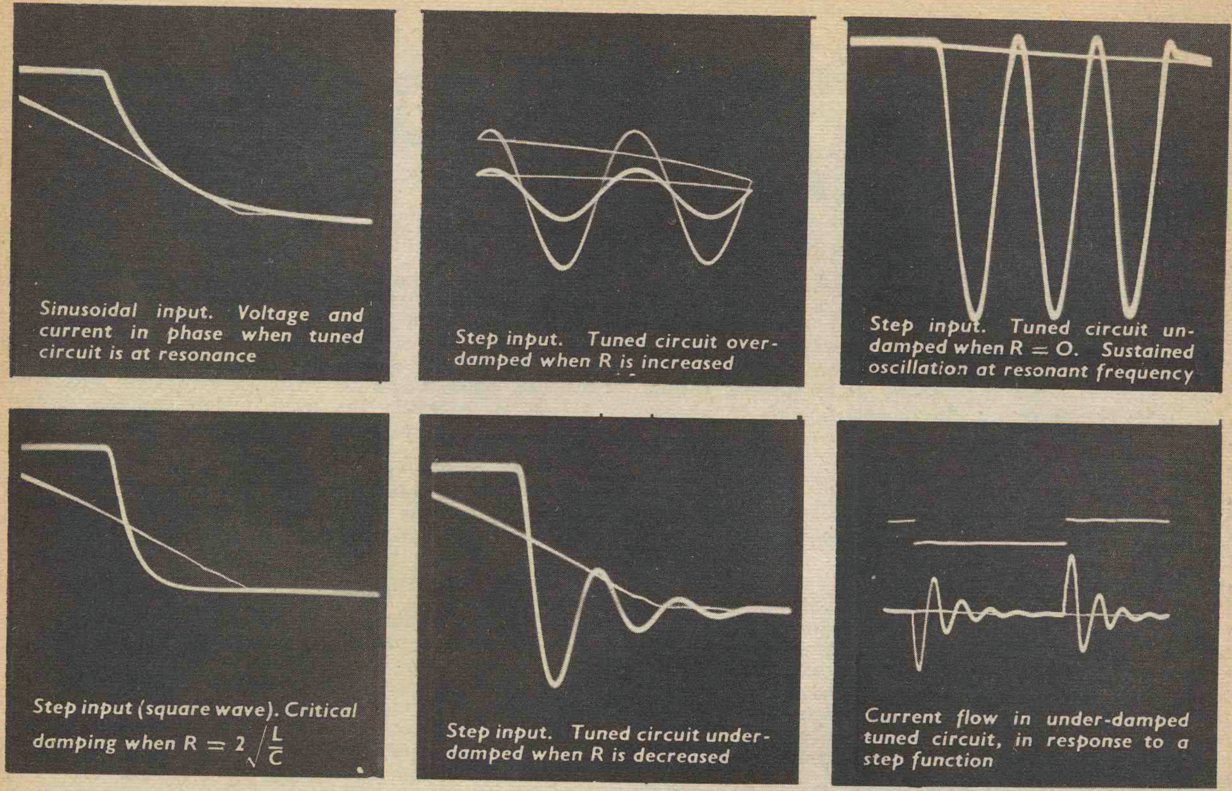


Fig. 5.7. Response of a simulated tuned circuit

Apply a 2V r.m.s. 50Hz signal to OA1/Input 2, and monitor by means of a reliable 10V a.c. meter of not less than 1 kilohm/volt sensitivity. The input function should preferably come from a low impedance source to avoid serious loading errors when the voltmeter is removed. Next, connect the a.c. voltmeter to the output of OA1 and adjust CP1 so that OA1 input and output voltages are exactly equal. CP1 could alternatively be set by the reference voltage and d.c. voltmeter method mentioned earlier, for a coefficient of 0·1. If the CP2 setting is altered it will be discovered that the simulated circuit goes off resonance, and can be tuned by CP2 between approximately 5Hz and 50Hz.

UNIT "A" will now be ready for analysis of the Fig. 5.6a tuned circuit, and will also cover a useful range of other values for *L*, *C*, and *R* in real time.

When handling sinusoidal or step functions, an amplifier will still have a maximum output voltage swing of ±10V, but this will be the peak voltage value. To check for overloading with an a.c. meter, ensure that amplifier output voltages do not exceed 7·07V r.m.s. for a sine wave function, and 5V mean for an equal mark-space square wave.

RESCALING PROBLEM EXAMPLE 3.

To rescale the problem for larger or smaller values of *L* and *C*, beyond the coverage of CP2, and by abandoning real time operation, note that a tenfold increase in tuned circuit frequency corresponds to a hundredfold increase in $1/LC$. For most applications, where the series resistance *R* will lie between zero and just beyond critical damping ($R > 2\sqrt{[L/C]}$), the scaling of R/L can stay as it is for all reasonable values of *L* and *C*, but should anyway only be changed by adjustment of the gain factor at OA1/Input 1. Similarly, the $f(t)/L$ gain of 100 at OA1/Input 2 can remain fixed.

It is not necessary to use inconveniently large or small input functions when rescaling for new voltages and currents. 2V r.m.s. could equally well represent an input function of, say, 0·2V r.m.s., and from Ohm's Law the current *I* will automatically become 2mA, instead of the former 20mA, even though it is still represented by 2 computer volts.

If it is desired to extend the computer operating time, by adjustment of integrator and inverting amplifier closed-loop gains, refer to Table 5.2, while remembering that integrator closed-loop gains are calculated on the basis of $1/R_{in}C_f$ where *R* is in ohms and *C* is in farads.

For reasons of reduced accuracy, it is not advisable to use computer operating frequencies above 1kHz or below 0·05Hz in connection with Problem Example 3. It should be mentioned that although frequencies in the region of 0·05Hz are too low for display on an a.c. coupled oscilloscope, the behaviour of a system can be demonstrated in slow motion by the oscillating movement of a d.c. voltmeter pointer (centre-zero).

Some typical oscillograms are given in Fig. 5.7 to show the response of a simulated tuned circuit. If the computer oscilloscope is provided with a good graticule, and has a linear response, amplitude and time measurements which are accurate to within approximately 5 per cent may be obtained straight from the trace.

The behaviour of a real tuned circuit can be evaluated by comparison with a simulated circuit. A tracing is made of the real circuit oscilloscope display, and is then superimposed on the readout given by the simulated circuit. The computer is adjusted so that time scales are related by a known factor, and tracing and readout display are identical, then quantitative measurements are taken from the computer voltages and dial settings.

Next month: The construction and operation of UNIT "B"